1903MNRAS..63..164G

A sufficient number of observations have not been made to test azimuth results, but such as they are they appear satisfactory. The only rough and ready method that occurred to me was to take the instrumental error in azimuth for each night as deduced from prime vertical and latitude stars respectively. Only two sets are available, as it did not occur to me to try this method for azimuth until the 12th, and even then I adopted different tactics for the two sets. For the prime vertical stars I kept the star bisected on the middle vertical wire with the slow-motion screw in azimuth until it reached a fixed horizontal wire, and took no time transits at all, reducing by a different method. I did not like this way, however, as well as the time transit, and only used it on that one occasion; but it is satisfactory to note the agreement between the two.

		P. Vert. Stars.	Latitude Stars.
Dec. 12	Inst. error in az.	+10″·2	+10″·7
13	" " "	+ 4·8	+ 4·8

Of course the change between the 12th and 13th is of no account. That was almost certainly a real change.

In conclusion I must express my great obligation to S. C. Chandler for his work on the Almucantar published as vol. xvii. of the *Annals of Harvard College Observatory*, to which I refer all those who wish to see more of the mathematics and results of this beautiful method.

On the Sun's Stellar Magnitude and the Parallax of Binary Stars. By J. E. Gore, M.R.I.A.

The Sun's stellar magnitude, or the number which represents the Sun's brightness as seen from the Earth on the same scale as the "magnitudes" of the stars are represented, has been variously estimated at numbers ranging from −25·5 to −27·0. The following method of computing its value from the apparent brightness of binary stars, whose orbits are well determined and whose spectra resembles that of the Sun, has not, so far as I am aware, been previously published.

Let P = period of binary star in years
a = semi-axis major of orbit in seconds of arc
p = parallax of binary

Then we have

$$\text{Mass of binary system} = \frac{a^3}{p^3 P^2} \quad (\text{Sun's mass} = 1)$$

1903MNRAS..63..164G

Now if we assume the binary pair to have the same density and surface brilliancy as the Sun, we have surface of single body equivalent in mass to the binary pair

$$= \left(\frac{a^3}{p^3\,\mathrm{P}^2}\right)^{\frac{2}{3}} = \frac{a^2}{p^2\,\mathrm{P}^{\frac{4}{3}}}$$

Then, as the relative brightness is proportional to the illuminated surface, we have

$$\text{Brightness of Sun : Brightness of binary} :: 1 : \frac{a^2}{p^2\,\mathrm{P}^{\frac{4}{3}}}$$

and

$$\therefore \text{ Brightness of Sun} = \frac{p^2\,\mathrm{P}^{\frac{4}{3}}}{a^2} \times \text{Brightness of binary} = \mathrm{B} \text{ (say).}$$

Now if m = photometric magnitude of binary, we have

$$m - \frac{\log \mathrm{B}}{0{\cdot}4} = \text{magnitude of Sun at distance of binary} = \mathrm{M}$$

$$(0{\cdot}4 = \log \text{“light ratio”} = \log 2{\cdot}512)$$

or

$$m - \left\{\frac{2\log p + \frac{4}{3}\log \mathrm{P} - 2\log a}{0{\cdot}4}\right\} = \mathrm{M}$$

or

$$m - 5\log p - \tfrac{10}{3}\log \mathrm{P} + 5\log a = \mathrm{M} \;\ldots\quad \ldots \quad (1)$$

And we have also Sun's magnitude at distance of binary (if S = Sun's stellar magnitude)

$$\mathrm{M} = \frac{\log\left(\frac{206265}{p}\right)^2}{0{\cdot}4} + \mathrm{S} \text{ (S being negative)}$$

$$= 5\log 206265 - 5\log p + \mathrm{S}$$

or

$$\mathrm{M} = 26{\cdot}57 - 5\log p + \mathrm{S} \;\ldots\quad \ldots\quad \ldots\quad \ldots \quad (2)$$

Equating (1) and (2), we have

$$m - 5\log p - \tfrac{10}{3}\log \mathrm{P} + 5\log a = 26{\cdot}57 - 5\log p + \mathrm{S}$$

whence

$$\mathrm{S} = m - 26{\cdot}57 - \tfrac{16}{3}\log \mathrm{P} + 5\log a \quad \ldots \quad (3)$$

This equation depends only on m, the photometric magnitude of the binary star, P its period, and a the semi-axis major of the computed orbit, and is independent of the star's parallax or distance from the Earth.

As, however, the combined surface of a pair of spheres is greater than that of a single sphere having the same mass, equation (3) will require a small correction. The amount of this correction is easily found. For let R = radius of single sphere, and r and nr = radii of two spheres having a combined mass

1903MNRAS..63..164G

equal to the single sphere. Then surface of single sphere is proportional to R^2, and combined surface of the two spheres is proportional to $n^2 + n^2r^2 = (n^2+1)r^2$; and we have also

$$R^3 = n^3r^3 + r^3 = (n^3+1)r^3$$

and

$$\therefore R^2 = (n^3+1)^{\frac{2}{3}} r^2$$

and ratio of combined surfaces of the two spheres to surface of single sphere is

$$(n^2+1)r^2 : R^2$$

or

$$(n^2+1)r^2 : (n^3+1)^{\frac{2}{3}} r^2$$

or

$$n^2+1 : (n^3+1)^{\frac{2}{3}}$$

If the components of the binary star are equal in size the ratio is $2 : 2^{\frac{2}{3}}$, or $2^{\frac{1}{3}} : 1$, or $1{\cdot}26 : 1$.

If $n = 2$, the ratio is $5 : 9^{\frac{2}{3}}$, or $1{\cdot}15 : 1$. This corresponds to a difference of about $1\frac{1}{2}$ magnitude between the components.

If $n = 4$, the ratio is $17 : 16{\cdot}166$. This corresponds to a difference of about 3 magnitudes between the components.

The correction is then made by adding the logarithm of this ratio divided by 0·4 to equation (3). For the extreme case, when the components are equal in size, the equation becomes

$$S = m - 26{\cdot}57 - \tfrac{10}{3}\log P + 5\log a + \frac{\log 1{\cdot}26}{0{\cdot}4}$$

or

$$S = m - 26{\cdot}57 - \tfrac{10}{3}\log P + 5\log a + 0{\cdot}25$$

or

$$S = m - 26{\cdot}32 - \tfrac{10}{3}\log P + 5\log a \qquad \ldots \qquad \ldots \qquad (4)$$

Let us now apply the formula to some examples of binary stars which have a spectrum closely resembling the solar spectrum, and of which the density and surface brilliancy are probably nearly the same as the Sun's.

α Centauri. $P = 81{\cdot}1$ years, $a = 17''{\cdot}7$ (See). Photometric magnitude = 0·20 (mags. of components 0·50 and 1·75, Pickering). Spectrum of solar type. Here we have

$$\begin{aligned} S &= 0{\cdot}20 - 26{\cdot}57 - \tfrac{10}{3}\log 81{\cdot}1 + 5\log 17{\cdot}7 \\ &= 0{\cdot}20 - 26{\cdot}57 - 6{\cdot}3634 + 6{\cdot}2398 \\ &= -26{\cdot}49 \end{aligned}$$

In this case the difference between the components being 1·25 magnitude, the correction comes out +0·18, and hence

$$S = -26{\cdot}49 + 0{\cdot}18 = -26{\cdot}31$$

ξ Ursæ Majoris. $P = 60$ years, $a = 2''{\cdot}508$ (See). Mag.

1903MNRAS..63..164G

= 3·93 (components 4, $4\frac{1}{2}$). Spectrum solar type (G. Pickering). Here we have

$$\begin{aligned} S &= 3{\cdot}93 - 26{\cdot}57 - \tfrac{10}{3}\log 60 + 5\log 2{\cdot}508 + 0{\cdot}23 \\ &= 3{\cdot}93 - 26{\cdot}57 - 5{\cdot}927 + 1{\cdot}996 + 0{\cdot}23 \\ &= -26{\cdot}34 \end{aligned}$$

η Cassiopeiæ. P = 500 years, $a = 11''{\cdot}4$ (Comstock). Mag. = 3·63 (components 4 and 7·6). Spectrum second type, F 8 G (Pickering).

Here we have

$$\begin{aligned} S &= 3{\cdot}63 - 26{\cdot}57 - \tfrac{10}{3}\log 500 + 5\log 11{\cdot}4 \\ &= 3{\cdot}63 - 26{\cdot}57 - 8{\cdot}99 + 5{\cdot}28 \\ &= -26\,65 \end{aligned}$$

In this case, as there is a difference of 3·6 magnitudes between the components, the correction is very small and may be neglected.

As the parallax of *α Centauri* is known (0″·75), we can compute the Sun's stellar magnitude from it in another way. The mass of the brighter component is about equal to the Sun's mass, and the spectrum very similar (G. Pickering); we can therefore fairly compare the two bodies.

Let p = parallax of the star.

Then $\dfrac{206265}{p} \times 93000000$ = distance of star.

Let θ = apparent diameter of the star in seconds of arc.

Then $\theta : 206265 :: 866000 :$ distance

(866000 = Sun's diameter in miles)

and $\therefore$

$$\theta = \frac{206265 \times 866000}{\text{distance}}$$

$$= \frac{206265 \times 866000}{\dfrac{206265}{p} \times 93000000}$$

or

$$\theta = \frac{866000p}{93000000}$$

$$= \frac{p}{107} \text{ or } = 0''{\cdot}009345p$$

For *α Centauri* we have

$$\theta = 0{\cdot}009345 \times 0{\cdot}75 = 0''{\cdot}007$$

and

$$\text{Brightness of sun} = \frac{(1920'')^2}{(0''{\cdot}007)^2} = \frac{3686400}{0{\cdot}000049} = 75232650000$$

1903MNRAS..63..164G

or the Sun is 75,232,650,000 times brighter than the brighter component of *α Centauri.*

Hence if the Sun and the brighter component of *α Centauri* have the same intrinsic brightness of surface and the same density, we have

$$(2{\cdot}512)^n = 75232650000$$

and $$n = 27{\cdot}2$$

or the Sun is 27·2 magnitudes brighter than the brighter component of *α Centauri.* And as the photometric magnitude of the star is 0·50, we have

$$\text{Sun's stellar mag.} = (0{\cdot}50 - 27{\cdot}2) = -26{\cdot}70$$

The mean of the four results found above is

$$S = -26{\cdot}50$$

Substituting this value of S in equation (2), we have

$$\begin{aligned} M &= 26{\cdot}57 - 5\log p - 26{\cdot}50 \\ &= -5\log p + 0{\cdot}07 \\ &= 5\log\left(\frac{1}{p}\right) + 0{\cdot}07 \quad \ldots \quad \ldots \quad \ldots \quad (5) \end{aligned}$$

a simple formula for obtaining the Sun's magnitude if removed to the distance of a star whose parallax is p.

In the case of a spectroscopic binary star having a solar spectrum, if we assume that the plane of the orbit passes through the Earth, or nearly so, we can find from the observed "radial velocity" the dimensions of the system in miles, and the combined mass of the components. From these data we can compute the probable parallax of the star in the following way:

Let μ = mass of spectroscopic binary

and m = its photometric magnitude.

Then from equation (5) we have (neglecting the small constant)

$$\left(5\log\frac{1}{p} - m\right) \times 0{\cdot}4 = \log \mu^{\frac{2}{3}}$$

or, $$2\log\frac{1}{p} - 0{\cdot}4m = \frac{2}{3}\log\mu$$

and $$\therefore \log\frac{1}{p} = \frac{1}{3}\log\mu + 0{\cdot}2m \quad \ldots \quad \ldots \quad (6)$$

Or, to obtain the parallax directly from the period and com-

1903MNRAS..63..164G

puted semi-axis major in miles $= d$, we have, if $R =$ radius of Earth's orbit,

$$\mu = \frac{\left(\frac{d}{R}\right)^3}{P^2}$$

and $$\therefore \log \mu = 3 \log d - 3 \log R - 2 \log P$$

Substituting this in equation (6), we obtain

$$\log p = -\tfrac{1}{3}(3 \log d - 3 \log R - 2 \log P) - 0{\cdot}2m$$

or
$$\begin{aligned} \log p &= \tfrac{2}{3} \log P - 0{\cdot}2m + \log R - \log d \\ &= \tfrac{2}{3} \log P - 0{\cdot}2m + \log (92800000) - \log d \\ &= \tfrac{2}{3} \log P - 0{\cdot}2m + [7{\cdot}9675480] - \log d \qquad \ldots \quad (7) \end{aligned}$$

Take the case of *η Pegasi*, which is a spectroscopic binary with a spectrum of the solar type (G. Pickering). The magnitude is 3·20. The spectroscopic observations show that the period is 818 days, and the semi-axis major of the orbit about 200,000,000 miles. This gives a mass about twice the Sun's mass. Hence from equation (6) we have

$$\begin{aligned} \log \frac{1}{p} &= \frac{1}{3} \log 2 + 0{\cdot}2 \times 3{\cdot}20 \\ &= 0{\cdot}10034 + 0{\cdot}64 \\ &= 0{\cdot}74034 \end{aligned}$$

$$\therefore \frac{1}{p} = 5{\cdot}5, \text{ and } p = \frac{1}{5{\cdot}5} = 0''{\cdot}182$$

From equation (7) we have

$$\begin{aligned} \log p &= \tfrac{2}{3} \log \tfrac{818}{365} + 7{\cdot}9675480 - \log 200000000 - 0{\cdot}64 \\ &= 0{\cdot}2334986 + 7{\cdot}9675480 - 8{\cdot}3010300 - 0{\cdot}64 \end{aligned}$$

or $$\log p = \bar{1}{\cdot}2600166$$

and $p = 0''{\cdot}182$, as before.

With this parallax I find, from equation (5), that the Sun would be reduced to a star of 3·77 magnitude if removed to the distance of *η Pegasi*. This makes the star 0·57 mag. brighter than the Sun, and its mass about 2·2 times the Sun's mass; a result which agrees closely with the mass derived from the spectroscopic observations.

In the case of spectroscopic binaries which have a spectrum of the *Sirian* type their light is not comparable with that of the Sun, and the above formulæ cannot be used. But we can compare them with *Sirius*, of which the parallax and mass are known, and thus deduce a probable parallax.

1903MNRAS..63..164G

Taking the parallax of *Sirius* at 0″·38 and the mass of the bright star, as deduced from See's orbit,* at 2·36, let p = parallax of binary, and μ its mass computed from the spectroscopic observations. Then we have ratio of light of *Sirius* to light of spectroscopic binary

$$= \left(\frac{0{\cdot}38^2}{p^2}\right) \times \left(\frac{2{\cdot}3\ 6}{\mu}\right)^{\frac{2}{3}}$$

and if m = magnitude of spectroscopic binary, we have, taking the magnitude of *Sirius* = − 1·62

$$\log \text{ ratio of light} = \{m - (-1{\cdot}62)\} \times 0{\cdot}4 = 0{\cdot}4m + 0{\cdot}648$$

Hence

$$\log \left\{ \left(\frac{0{\cdot}38}{p}\right)^2 \times \left(\frac{2{\cdot}36}{\mu}\right)^{\frac{2}{3}} \right\} = 0{\cdot}4m + 0{\cdot}648$$

or

$$2 \log 0{\cdot}38 + \tfrac{2}{3} \log 2{\cdot}36 - 2 \log p - \tfrac{2}{3} \log \mu = 0{\cdot}4 \text{ m} + 0{\cdot}648$$

or

$$\bar{1}{\cdot}1595 + 0{\cdot}2486 - 0{\cdot}648 - 2 \log p - \tfrac{2}{3} \log \mu = 0{\cdot}4m$$

whence

$$\log p = \bar{1}{\cdot}380 - \tfrac{1}{3} \log \mu - 0{\cdot}2\, m \qquad \ldots \quad \ldots \qquad (8)$$

Let us take some examples of the application of this formula.

Algol. Vogel finds the mass of the bright component of this well-known variable star to be $\frac{4}{9}$th of the Sun's mass = 0·4444 × Sun.

Its photometric magnitude at normal brightness as measured at Harvard is 2·31. Hence we have

$$\begin{aligned} \log p &= \bar{1}{\cdot}380 - \tfrac{1}{3} \log 0{\cdot}4444 - 0{\cdot}2 \times 2{\cdot}31 \\ &= \bar{1}{\cdot}380 - \bar{1}{\cdot}8826 - 0{\cdot}462 \\ &= \bar{1}{\cdot}035 \end{aligned}$$

and $p = 0''{\cdot}1082$

Chandler computed a parallax of 0″·07.

Spica. Mag. 1·09. Vogel found from the spectroscopic observations a combined mass of 2·6 × Sun's mass. Assuming the bright and dark components to be equal in mass, we have mass of bright component = 1·3, and hence :

$$\begin{aligned} \log p &= \bar{1}{\cdot}380 - 0{\cdot}0379 - 0{\cdot}218 \\ &= \bar{1}{\cdot}1241, \text{ and } p = 0''{\cdot}133 \end{aligned}$$

As the star is not an *Algol* variable, the plane of the orbit

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* *Researches on the Evolution of the Stellar Systems*, vol. i. p. 85.

1903MNRAS..63..164G

does not probably pass through the Earth. Hence the mass is probably larger, and the parallax smaller, than that found above. No parallax has been found for this star.

β *Aurigæ.* Mag. 1·98. Computed mass = 5·5 × Sun's mass. In this case both components are bright, and Pickering says: "Both components appear to be equal, or nearly equal, in brightness and to have similar spectra (Spectrum A). We must therefore deduct from the constant in equation (8), $\frac{1}{2}\log(1{\cdot}26)$, or 0·05, and the formula becomes

$$\log p = \bar{1}{\cdot}330 - \tfrac{1}{3}\log\mu - 0{\cdot}2m$$

$$\begin{aligned}\text{or } \log p &= \bar{1}{\cdot}330 - \tfrac{1}{3}\log 5{\cdot}5 - 0{\cdot}2 \times 1{\cdot}98\\ &= \bar{1}{\cdot}330 - 0{\cdot}2467 - 0{\cdot}396\\ &= \bar{2}{\cdot}6873, \text{ and } \therefore p = 0''{\cdot}04868\end{aligned}$$

The late Professor Pritchard found by means of photography a parallax of 0″·065 and 0″·059.

ζ *Ursæ Majoris.* Mag. 2·18. (Both components bright.) Period according to Vogel and Eberhard 20·6 days, and mass $= \dfrac{4 \times \text{Sun's mass}}{\sin^3 i}$.

If we suppose $i = 90^\circ$, $\sin^3 i = 1$, and the mass will be 4 × Sun's mass. Supposing the components to be equal in brightness, we can use the same formula as for β *Aurigæ*, and

$$\log p = \bar{1}{\cdot}330 - \tfrac{1}{3}\log 4 - 0{\cdot}2 \times 2{\cdot}18$$

This gives $\log p = \bar{2}{\cdot}6933$, and $\therefore p = 0''{\cdot}049$

Höffler found a parallax of 0″·0165, but a parallax of 0″·045 was found by Klinkerfues.

Castor. Mag. 1·61 (components about 1·98 and 2·98). The brighter component of this well-known binary star has been found to be a spectroscopic binary, the companion being a dark body like *Algol.* Period 2·98 days. Orbital velocity = 20·7 miles a second. If we suppose, as in the case of *Algol*, that the bright component has twice the mass of the dark companion, I find that a = 2,550,000 miles (= 1,700,000 × $\frac{3}{2}$), and the mass = 0·3113 × Sun's mass. Hence mass of bright component = 0·3113 × $\frac{2}{3}$ = 0·2075, and we have

$$\begin{aligned}\log p &= \bar{1}{\cdot}380 - \tfrac{1}{3}\log(0{\cdot}2075) - 0{\cdot}2 \times 1{\cdot}98\\ &= \bar{1}{\cdot}2117 \text{ and } \therefore p = 0''{\cdot}1628\end{aligned}$$

From heliometer measures of *Castor* made by Johnson at Oxford in 1854–55 he found a parallax of 0″·198.

The parallaxes found above for spectroscopic binaries have been computed on the assumption that the plane of the orbit passes through the Sun, or nearly so. If the orbital plane is inclined to the line of sight—as it probably is in most cases—

1903MNRAS..63..164G

the mass of the system will be greater and the parallax consequently less. If the data used are correct, the computed values are therefore a maximum.

With reference to stars having spectra between the second and third types, the star λ *Andromedæ* is, I find, an interesting case. The spectroscopic observations show a period of about 19·2 days and an orbital velocity of about 5·6 miles a second. From this I find (supposing the orbit plane to be in the line of sight) a mass of only 0·0119 × Sun's mass. As the star's spectrum is K, according to Pickering it is not comparable with the Sun, and the formulæ given above cannot be used in this case; but as its photometric magnitude is 4·14, the very small mass would suggest that the star is comparatively near the Earth. If we suppose the inclination of the orbit to be 30°, the mass would be increased eight times; but even then the mass would be less than one-tenth of the Sun's mass.

Dublin: 1903 *January* 4.

New Double Stars detected with the 17¼-*inch reflector during the year* 1902. By T. E. Espin.

The following list contains new double stars found during 1902. The weather has been continuously bad throughout the year, and even when the sky has been clear the definition has generally been very poor. Many of the *comites* are far too faint to measure satisfactorily.

No.	B.D.	R.A. 1880.		Decl.		P.	D.	Night.	Mags.		Note.
	°	h	m	°	′	°	″				
113	66·6	0	2·6	+66	37	122·6	6·8	1	8·5	11	...
114	66·7		3·1	66	29	161·6	5·0	3	8·7	11·2	...
115	61·50		16·3	61	34	82·4	9·9	1	8·0	10	...
116	54·87		24·6	54	59	255·9	7·7	1	8·9	8·9	...
117	54·106		27·6	55	3	54·4	3·0	1	9·0	11	...
118	63·111		47·7	63	43	241·3	2·6	2	8·6	8·7	...
119	53·271	1	10·3	54	19	115·1	5·1	1	8·2	10·5	...
120	53·576	2	41·1	53	26	70·3	3·9	1	8·7	12·5	...
121	57·729	3	24·1	57	51	325·5	6·9	2	8·0	13·5	...
122	61·666		55·3	61	50	248·7	5·0	2	8·6	10·5	...
123	44·2120	11	37·4	44	51	203·8	7·4	1	9·1	9·3	AB.
						275·5	42·9	1		9·2	AC.
124	42·2287	12	10·2	42	34	135 ±	5 ±	1	9·0	12·5	Too faint.
125	42·2370		59·9	42	19	119·1	2·4	2	8·0	10·6	...